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AN OVERVIEW OF SIGNAL PROCESSING FOR ARRAYS OF RECEIVERS

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An Overview of Signal Processing for Arrays of Receivers

H.A. d'Assumpcao* and G.C. Mountford**

An Invited Paper

1 INTRODUCTION

Arrays of receivers are used in many fields - in radio, radar, sonar, seismic exploration, ultrasonic diagnosis, and so forth - to detect weak signals, to resolve closely-spaced targets and to estimate the bearing and other properties of a signal source. This tutorial paper is concerned only with the theory of receiving arrays, which is common across several fields. A limited theoretical development is first presented to introduce the mathematical basis and the terminology of the topic. Then a brief review is made of several array processing techniques with some discussion of theory and performance. No attempt is made to cover practical implementations, because the technologies involved can differ widely between one application and another. For example, radar systems with very wide bandwidth and which demand real-time outputs are unable at present to use certain advanced processing techniques; on the other hand, in some seismic exploration applications, it may be acceptable for results to be available some time after the measurements are taken, so time-consuming computation is less of a problem.

In all that follows, we consider only idealised arrays - i.e., those in which there are no cross-couplings between individual receivers. The receivers are assumed to sample the spatial field without distorting it. We further assume that the spatial field in the vicinity of the array is homogeneous (an assumption that is often valid when all signal sources are distant from the array) and that all receivers have the same sensitivity. (If there are variations in the responses of receivers, the changes needed in the theory are usually trivial.)

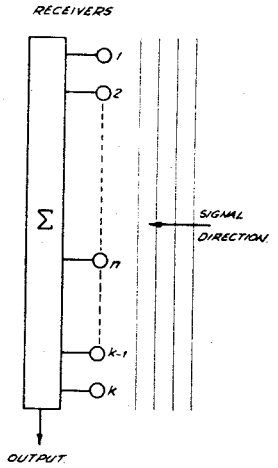
In section 2 to 4 we consider beamformers of various kinds. Developments in estimation are addressed in section 5. The application of some important spectral analysis techniques to array processing is outlined in section 6. Then in section 7 adaptive processing techniques are covered briefly. Finally, some discussion is given in section 8.

2 CONVENTIONAL BEAMFORMER

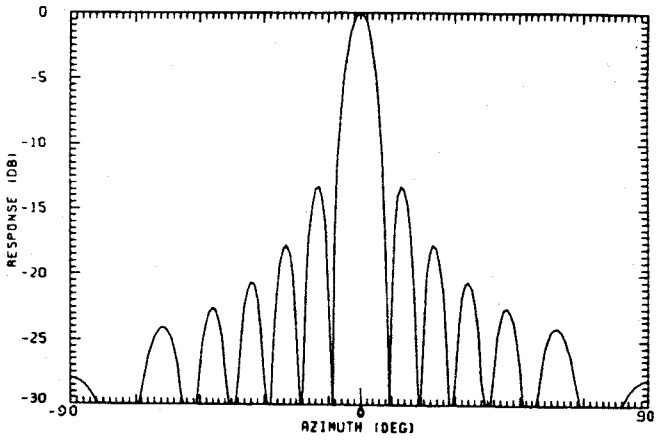
2.1 Beamforming

The principles of conventional beamforming were established many decades ago. Consider a linear array, as shown in fig. 1(a). A plane wavefront arriving from a direction normal to the array (i.e., broadside) arrives at all the receivers in phase, and so if summed the signal outputs from the receivers will add in phase and reinforce one another; signals from other directions will not be in phase, and so will not be reinforced.

If it is desired to receive narrowband signals from some other direction than broadside, we simply shift the phases of the receiver outputs by appropriate amounts to bring all the receiver outputs into phase before summing (fig. 2(a)). More generally, if the signals are broadband, we time-delay the receiver outputs appropriately before



(a)



(b)

Figure 1 Broadside linear array. A signal from broadside is enhanced by summation.

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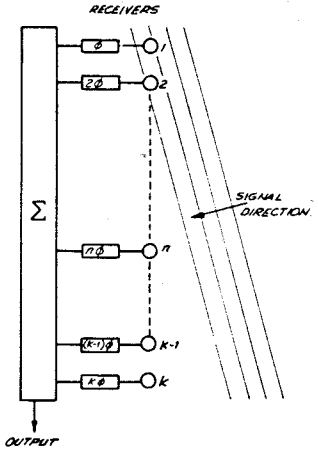
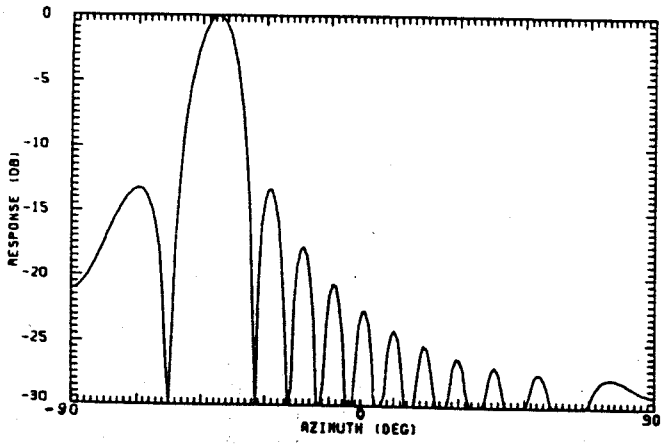
summation. This process is called 'beamsteering'. It can readily be seen that the same principles can be extended to arrays of arbitrary geometry in two or three dimensions: we simply insert the appropriate time delays, as illustrated in fig. 3. Most of the theoretical development in this paper will relate to narrowband processing in the frequency domain.

It is evident that the technique is not confined to plane wavefronts: curved wavefronts may similarly be catered for. However, in this paper we consider only plane wavefronts.

If we plot the average power output as a function of signal direction, we obtain the polar diagram or polar response, as shown in figs. 1(b) and 2(b). By convention, the polar response is normalised so that its response in the beamsteered direction is unity. In our diagrams, it is plotted in dB relative to the value in the steered direction. The beamwidth (i.e., the width of the main lobe, often defined as the angle between the points at which the power response falls by half) indicates the ability to resolve closely-spaced signal sources; the level of the sidelobes is a measure of susceptibility to interference by unwanted signals in directions away from the main beam. We shall derive an expression for the polar response later in this section.



(a)



(b)

Figure 2 Beamsteered linear array (narrowband signals). An off-broadside signal is enhanced by phase-shifting the receiver outputs so that signals add in phase.

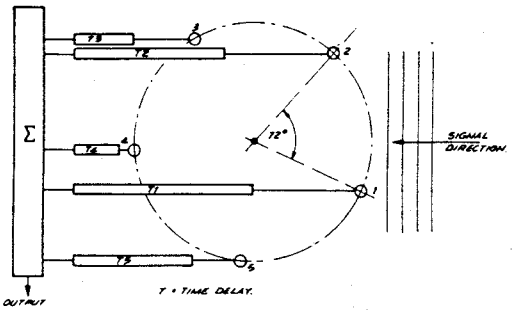


Figure 3 Time domain beamsteering (broadband signals). Receiver outputs from the direction of the wanted signal are delayed to bring them into coincidence, and are then added.

2.2 Wavenumber Representation

Consider the simple case of a linear array of K receivers, equally spaced apart by a distance d. As stated previously, in the remainder of this paper we assume that all signals and noises are narrowband. We select a system of coordinates as illustrated in fig. 4[1]. Because of rotational symmetry, we are unable to distinguish between signals arriving at the same angle θ to the axis of the array. For example we cannot deduce whether a signal is arriving from θ or (π - θ), so we need only consider angles between ±π/2.

However, instead of working with angles, it is convenient to use wavenumber k defined by

$$k = 2\pi/\lambda.$$

For a given signal direction, let

$$k' = (2\pi\sin\theta)/\lambda \qquad (1)$$

be the projection of k along the axis of the linear array,

where λ is the wavelength at the frequency of interest.

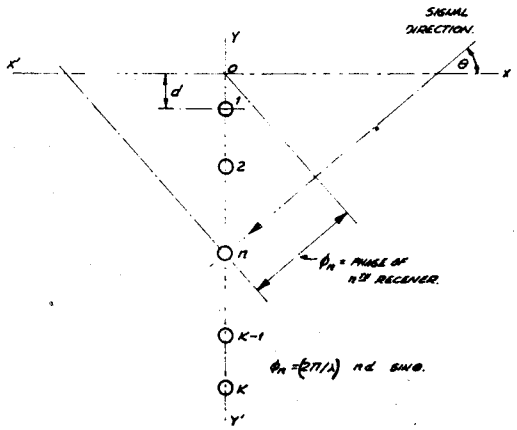


Figure 4 Linear array coordinate system. The array is unable to distinguish between signals from θ and (π - θ).

1 The origin is arbitrary and is chosen here for mathematical convenience.

It can be seen that the angular distribution of signals can be expressed in terms of k' rather than θ. Let y(k') denote the complex spectral amplitude of the signal as it arrives at the origin. The phase of that signal arriving at the n-th receiver is exp(ik'nd); in the absence of any internal system noise, the total signal received by the n-th receiver, from all external sources, is obtained simply by integration[2]:

$$x_n = \int_{-\infty}^{\infty} y(k') \exp(ik'nd)\, dk'. \qquad (2)$$

Equation 2 shows that x_n and y(k') are a Fourier transform pair. There is therefore a direct analogy between estimating a spectrum from a time series and estimating signal strengths from a set of spatial samples using a linear array of equispaced receivers. The wealth of techniques developed for the former case can therefore be applied directly to the latter. We shall address some of these later.

For a general three-dimensional array, instead of k' we use the vector $\underline{k}$.[3]

$$\underline{k} = \begin{Bmatrix} (2\pi \cos\theta \sin\phi)/\lambda \\ (2\pi \sin\theta \sin\phi)/\lambda \\ (2\pi \cos\phi)/\lambda \end{Bmatrix}, \qquad (3)$$

where θ, φ are horizontal and vertical angles, respectively. The signal at the n-th receiver is then

$$x_n = \int_{-\infty}^{\infty} y(\underline{k}) \exp(i\, \underline{k}.\underline{z}_n)\, d\underline{k}, \qquad (4)$$

where $\underline{z}_n$ is the co-ordinate vector of the n-th receiver and (.) denotes a dot product.

2.3 Polar Response

For a linear array, to steer a beam in a direction corresponding to a wavenumber projection k_o', we phase-shift the output of the n-th receiver by $k_o'nd$; the amplitude of the beamformer output is then

$$Y(k_o') = \sum_n x_n \exp(-ik_o'nd). \qquad (5)$$

The extension to an arbitrary array is straightforward:

$$Y(\underline{k}_o) = \sum_n x_n \exp(-i\, \underline{k}_o.\underline{z}_n). \qquad (6)$$

Unless stated to the contrary, the remainder of this paper applies to arrays of any geometry. Equation 6 can be expressed conveniently in matrix notation.

Let .* denote the complex conjugate of a matrix or vector,

.T its transpose,

.H = .*T the complex conjugate transpose, and

E{.} an expectation.

Let us define $\underline{x} = \{x_1 \ldots x_n \ldots x_K\}^T$,

and a vector of phase delays

$\underline{v}(\underline{k}_o) = \{v_1 \ldots v_n \ldots v_K\}^T$,

where $v_n = \exp(i\underline{k}_o.\underline{z}_n)$. (7)

For brevity, we shall write $\underline{v} \equiv \underline{v}(\underline{k}_o)$ and $v_n \equiv v_n(\underline{k}_o)$ (with the dependence on $\underline{k}_o$ being understood).

$\underline{v}$ is often called the 'steering vector', because when applied to the sensor signals it introduces the appropriate phases to steer a beam in the direction defined by $\underline{k}_o$.

The amplitude of the beamformer output is

$$Y(\underline{k}_o) = \underline{v}^H\underline{x}, \qquad (8)$$

and the array output power is

$$|Y(\underline{k}_o)|^2 = \underline{v}^H\underline{x}\underline{x}^H\underline{v}. \qquad (9)$$

It is often possible to consider the received signals to be stationary random processes with zero means[4]. In what follows we assume that to be the case. When a single plane wavefront arrives at the array, $\underline{x} = c\underline{u}(\underline{k}_s)$, where c is a zero-mean complex random variate with signal power $E\{c^Hc\} = \mu$, and $\underline{u} \equiv \underline{u}(\underline{k}_s)$ is the vector corresponding to the signal arrival direction ($\underline{k}_s$). $\underline{u}$ takes the same general form as $\underline{v}$:

$$\underline{u} = \{u_1 \ldots u_n \ldots u_K\}^T \qquad (10)$$

where $u_n = \exp(-i\underline{k}_s.\underline{z}_n)$.

We define $P(\underline{k}_o)$ to be the expected or average power out of the beamformer steered in direction implied by $\underline{k}_o$.

Then
$$\begin{aligned} P(\underline{k}_o) &= E\{|Y(\underline{k}_o)|^2\} \\ &= \underline{v}^H E(\underline{x}\underline{x}^H)\underline{v} \\ &= \mu|\underline{v}^H\underline{u}|^2. \end{aligned} \qquad (11)$$

When $\underline{k}_o = \underline{k}_s$, $\underline{u} = \underline{v}$ and $P(\underline{k}_s) = \mu K^2$. Apart from a scaling factor $1/(\mu K^2)$, equation 11 gives the polar response of the beamformer (as a function of $\underline{k}_s$).

2.4 Noise Response

In addition to discriminating between a wanted signal and unwanted signals (interference), the conventional beamformer also suppresses internal noise. Consider a simple conventional beamformer in which there is noise (e.g., receiver thermal noise) in the receivers. We assume that all the noises have the same power and that they are all statistically independent of one another. The wanted signal is brought into phase and summed, so its amplitude is increased K time (and hence its power by K^2). However, the noises, being

2 The limits of integration over k' have been set at ±∞, for generality. Of course, $|k'| \leq 2\pi/\lambda$ for planewave arrivals, but in practice it is sometimes necessary to include values of $|k'| > 2\pi/\lambda$. For example, in some sonars, signals are encountered which propagate along the mechanical structure of the array at a velocity less than that in the medium (water); such signals have a wavenumber such that $|k'| > 2\pi/\lambda$ and can legitimately be included.

3 We denote matrices and column vectors by underscored uppercase and lowercase letters, respectively.

4 Loosely speaking, they have no D.C. component and are noise-like with statistics that do not change with time.

independent, will sum with random phase, so their power is increased only K times. The resulting signal-to-noise ratio is therefore improved by a factor of K.

We now quantify the improvement in the more general case. In what follows, we use the terms "noise" and "interference" interchangeably, because the results apply to both internal noise and external interference, provided only that they add linearly to the signals at the receiver.

We define signal-to-noise power ratio in the usual way

$$\text{SNR} = \frac{\text{Change in power when signal present.}}{\text{Power due to noise only}} \quad (12)$$

The usual parameter used to quantify the performance of the array against noise is the array gain, defined as

$$G(\underline{u}) = \frac{\text{SNR output from beam steered at signal direction } \underline{k}_s.}{\text{SNR at single receiver}} \quad (13)$$

If noise and signal are all stationary random processes, and the noises are statistically independent of the signal, then the average beam output power is, as in equation 11,

$$P(\underline{k}_o) = E\{|Y(\underline{k}_o)|^2\} = \underline{v}^H(\mu \underline{M}_S + \nu \underline{M}_N)\underline{v} \quad (14)$$

where ν = noise power, which is here assumed to be the same for all receivers, and $\underline{M}_S$ and $\underline{M}_N$ are the normalised cross-spectral matrices for signal and noise respectively:

$E(\underline{x}\underline{x}^H) = \mu \underline{M}_S = \mu \underline{u}\underline{u}^H$ when only signal is present, and

$E(\underline{x}\underline{x}^H) = \nu \underline{M}_N$ for noise alone.

When $\underline{v} = \underline{u}$, i.e., when the beam is steered in the signal direction, we have

$$P(\underline{k}_s) = \mu K^2 + \nu \underline{u}^H \underline{M}_N \underline{u}. \quad (15)$$

When only noise is present, the beam output power in direction $\underline{k}_s$ is

$$P(\underline{k}_s) = \nu \underline{u}^H \underline{M}_N \underline{u}, \quad (16)$$

and the array gain is

$$G(\underline{u}) = K^2/(\underline{u}^H \underline{M}_N \underline{u}). \quad (17)$$

For the special case in which the noises at all the receivers are independent of one another, $\underline{M}_N = \underline{I}$ (the identity matrix) and $G(\underline{u}) = K$, as we have previously shown. As can be seen from equation 17, array gain depends on the covariance properties of the ambient noise. This is illustrated in fig. 5 for a simple circular array in spherically and cylindrically isotropic noise environments (i.e., with distant noise sources uniformly distributed over a sphere and over a circle, respectively).

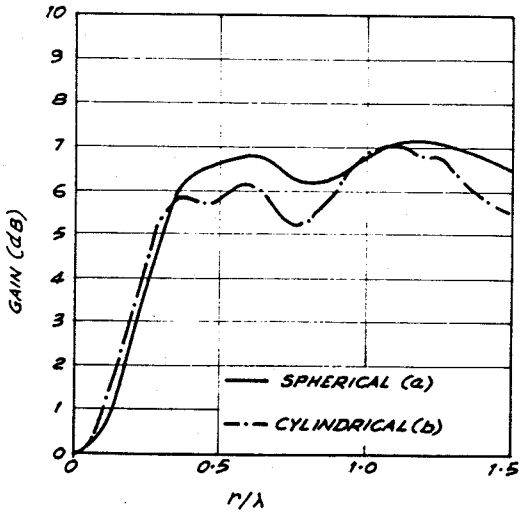
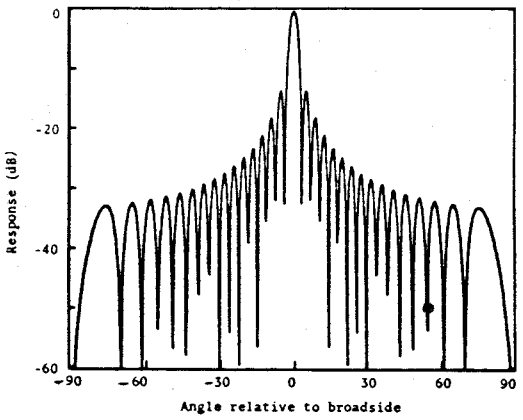


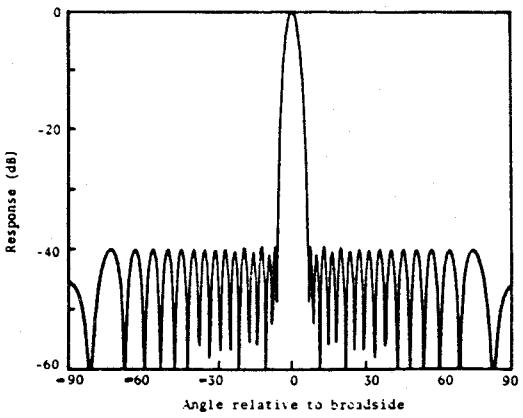
Figure 5 Effect of noise field on array gain. Simple five-element array of fig. 3, of radius r:
(a) in spherically isotropic noise, and
(b) in cylindrically isotropic noise.

2.5 Array Shading

Shading is a well-known technique used to modify the polar diagram of an array. In brief, instead of the steering vector $\underline{v}$ we use a weight vector $\underline{w}$,



(a) Conventional beamformer



(b) Dolph-Chebyshev shading

Figure 6 Polar diagrams illustrating effect of shading

where $\underline{w} = \{w_1 \ldots w_n \ldots w_K\}^T$

$w_n = a_n v_n$,

and a_n is real.

It has long been known that, by reducing the magnitude of the weights a_n as we move from the centre towards the edges of the array, the sidelobe levels are reduced, but at the cost of increasing the beamwidth[5].

Low sidelobes are usually desirable to reduce the effect of interfering signals. More complex design procedures, such as those used by Dolph [1] many years ago, permit a high degree of polar diagram control. Figure 6 gives an example of the result of Dolph-Chebyshev shading; the sidelobes all have the same peak level which has been selected to be much lower than the levels for the conventional beamformer.

Array output power is calculated using the same formula as for conventional beamforming, but with $\underline{v}$ replaced by $\underline{w}$ in equation 14.

2.6 Superdirectivity

It is sometimes desirable to use a given array at low frequencies - i.e., when the spacing between adjacent elements is appreciably less than half a wavelength. Conventional beamforming then yields a broad beam. However, by appropriate choice of the weight vector $\underline{w}$, it is possible to generate narrow beamwidths and the array is then called "super-directive". But there is a penalty: as the frequency is reduced, the magnitude of the weight required becomes large, making the practical implementation of superdirectivity difficult; the precision of computation is increased as is susceptibility to receiver noise. Examples of superdirective arrays are given later.

3 POLAR DIAGRAM CONTROL

We have described the technique of using real weights (shading) to reduce sidelobes or to narrow the main lobe at low frequencies (superdirectivity). It is possible to generalise and use complex weights to control the shape of the polar diagram. There are many different rationales for selection of techniques and some are briefly introduced in this section.

3.1 Minimum Bias

Another technique for reducing sidelobes is to select the set of weights which minimises the total volume under the polar diagram, while constraining the response in one direction (the 'beamsteered' or 'boresight' direction) to a value of unity. This process is called Minimum Bias Beamforming because it minimises the bias, imposed by noise sources in directions away from the main beam, on the estimate of signal strength [2, section V]; it tends to optimise performance in isotropic noise. A typical polar diagram is shown in figure 7.

3.2 Null-steering

Shading techniques and the minimum bias process described previously reduce sidelobes in general and are effective where interfering signal sources

5 Shading is in fact analogous to windowing in spectrum analysis.

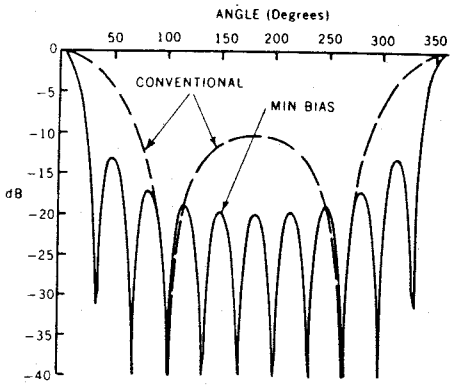


Figure 7 Theoretical polar diagram for minimum bias estimator. Ten-element circular array, $r/\lambda = 0.25$, where r = radius of array.

are smoothly distributed around the array. However, very often one is faced with interference (whether accidental or deliberate) from distant sources. It is then desirable to generate a polar diagram with nulls in the directions of the interfering sources.

One of the earliest successful techniques to generate a single null is called DICANNE [3,4], illustrated in fig. 8. The idea is basically simple: a beam steered in the direction of the interference is used to estimate its waveform; this estimate is subtracted from the receiver outputs.

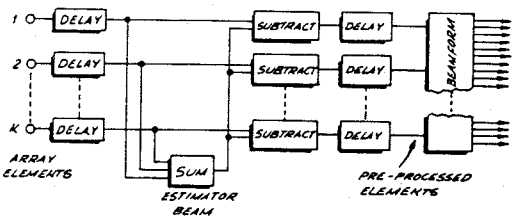


Figure 8 DICANNE beamformer to steer single null.

Finally beams are steered in the direction of the wanted signal in the usual way but using appropriately modified delays. It can be shown that, with a single strong interfering source, the performance of this process is similar to that of the optimal beamformer described in section 4.1.

DICANNE steers a single null. In general, for an array of K receivers, it is possible to generate nulls in (K-1) directions, while constraining the response in the desired direction to unity. In one method [2, section VIII], we first form a (KxK) matrix $\hat{\underline{R}}$ whose first column is the vector corresponding to the wanted signal, and whose remaining columns are the vectors corresponding to the (known) directions of the unwanted noises - that is, the directions of the K-1 nulls. In other words,

$$\hat{\underline{R}} = [\underline{u}\ \underline{v}_2 \cdots \underline{v}_K]. \qquad (18)$$

Next we invert $\hat{\underline{R}}$ and use its first row as our steering vector. It is evident that the response in the wanted direction will be unity and that the responses in the directions of all the (K-1) interferences will be zero. A typical polar diagram is

shown in fig. 9. This processor has been called the 'Maximum Likelihood Estimator' because it was originally derived by maximising the likelihood function. (This processing technique should not be confused with Capon's maximum likelihood method, described in section 4.2.)

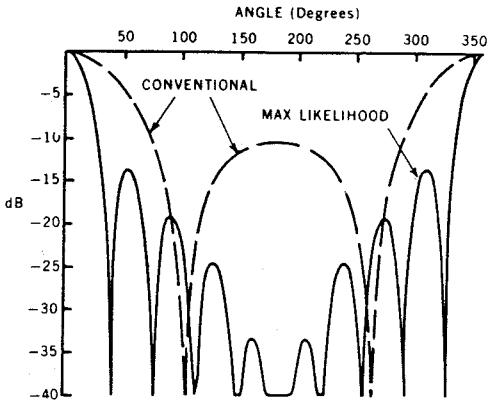


Figure 9 Polar diagram for beamformer steering (K-1) nulls. Ten-element circular array, $r/\lambda = 2.5$, where r = radius of array.

3.3 Bounds on the Polar Diagram

Null-steering techniques (such as those described in section 3.2) impose equality constraints – i.e., they force the response to be unity in the direction of the wanted signal, and zero in the directions of the interfering sources. However, it is often found that forcing the polar diagram to take a specific value in one direction causes severe distortions elsewhere. For example, if we attempt to steer a null too close to one side of the main lobe, the main lobe will be distorted on the other side; and if we try to steer two nulls, one on either side of the main lobe, the sidelobes elsewhere can increase to an unacceptable level, as illustrated in figs. 10 and 11 [5].

In practice, we often do not have to (nor, indeed, wish to) force equality, but need only specify upper and/or lower bounds on the polar diagram – in other words, we impose limits that are acceptable. Now, instead of solving a problem with equality constraints, we have inequality constraints. Evans [6] outlines some recent interesting work in which bounds are placed on the voltage response of an array. Separate inequality constraints are imposed on the real and imaginary parts of the voltage response. There would then in general be infinitely many sets of weights $\{w_1 \ldots w_K\}$, each of which would produce a voltage response meeting the constraints. It is natural to choose that set for which $\underline{w}^H\underline{w}$ is minimum, because large weights can introduce problems in computational accuracy and cause an increase in system noise. The problem then becomes one of quadratic minimisation subject to linear inequality constraints.

However, in most practical cases, one is interested, not in the voltage response of the array, but in its power response. We may then proceed to minimise the array weights, as before, but now impose limits on the array power response. The problem then becomes one of quadratic minimisation subject to quadratic inequality constraints – a far less tractable problem. One has to resort to a numerical solution which is rather cumbersome. However, preliminary

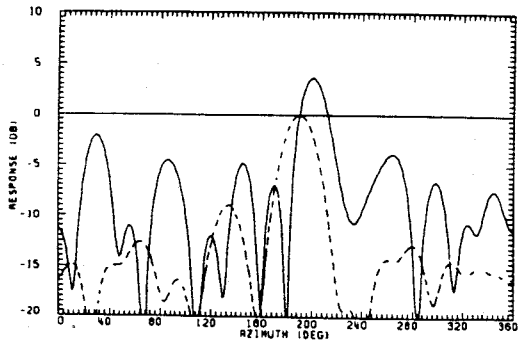


Figure 10 Illustrating effect of steering a null too close to the main lobe.
– – – – Conventional beamformer,
——— Null-steering processor.

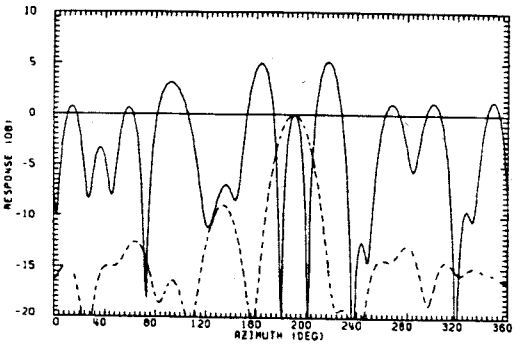


Figure 11 Illustrating effect of steering two nulls too close to the main lobe.
– – – Conventional beamformer,
——— Null-steering processor.

results [7] suggest that worthwhile improvements are possible. There is scope for further research work in this field.

4 OPTIMAL BEAMFORMING

In section 3, we considered techniques for generating nulls in the polar diagram, which are very effective when noise is due to a few unwanted plane wavefront arrivals whose directions are known. Here we address the problem of selecting a set of weights to maximise the signal-to-noise ratio for a more general case.

4.1 Unconstrained Optimal Beamforming

We make the same assumptions as before (i.e., the signal and noise are independent of one another, and both are zero-mean stationary random processes); in addition, we make the critical assumption that we know the cross-spectral matrix ($\underline{M}_N$) of the noise, and that $\underline{M}_N$ is nonsingular.

Let $\underline{w}$ be some general complex set of weights. The array gain is then

$$G(\underline{w}) = |\underline{w}^H\underline{u}|^2/(\underline{w}^H\underline{M}_N\underline{w}) \qquad (19)$$

as before.

This can be easily shown to be a maximum when

$$\underline{w} = \underline{M}_N^{-1}\underline{u} \qquad (20)$$

In other words, the set of weights which maximises the signal-to-noise ratio is obtained by premultiplying the signal arrival vector by the inverse of the noise cross-spectral matrix. Resubstituting equation 20 into equation 19, we find that the maximum gain is

$$G(\underline{M}^{-1}\underline{u}) = \underline{u}^H\underline{M}_N^{-1}\underline{u} \; . \qquad (21)$$

Applying the Kantorovich Inequality [8] we confirm that the gain of the optimal beamformer is always greater than or equal to that of conventional beamformer:

$$1 \leq G_{opt} \; / \; G_{conv}$$

$$\leq \left\{(\lambda_{max}/\lambda_{min})^{\frac{1}{2}} + (\lambda_{min}/\lambda_{max})^{\frac{1}{2}}\right\}^2/4, \qquad (22)$$

where G_{opt} and G_{conv} are the gains of the optimal and conventional beamformers, respectively, and λ_{max} and λ_{min} are the maximum and minimum eigenvalues of $\underline{M}_N$, respectively.

In practice, it is generally observed that the improvement of the optimal over the conventional beamformer is greatest when $\underline{M}_N$ is ill-conditioned

i.e., when $\lambda_{max} >> \lambda_{min}$. (23)

As mentioned earlier, the gain of a conventional beamformer reduces to K when the noises are independent (i.e., $\underline{M}_N = \underline{I}$); the optimum weights $\underline{w}$ under that condition also reduce to the steering vector $\underline{u}$ of the conventional beamformer (see equation 10).

Figure 12 illustrates the potential array gain improvement over a conventional processor, obtained

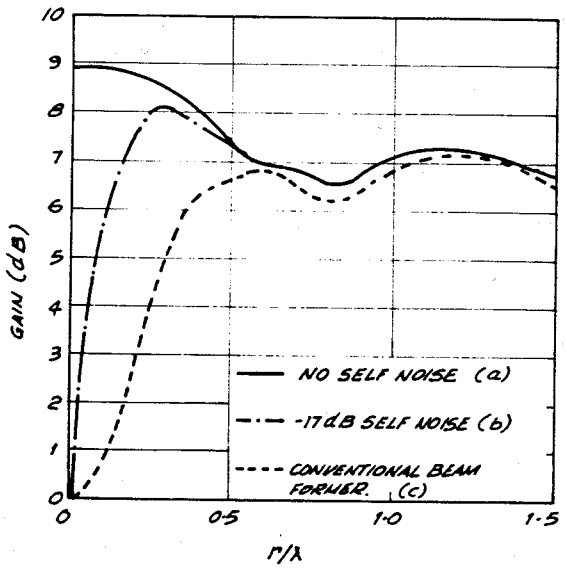


Figure 12 Array gain for optimal beamformer. Five-element circular array of fig. 3;
(a) gain for optimal beamformer with no uncorrelated self-noise,
(h) gain for optimal beambormer with receiver self-noise of -17 dB relative to ambient noise,
(c) gain for conventional beaformer.
Note the large improvement at low frequencies of the optimal over the conventional beamformer.

using the optimal processor in a spherically isotropic (i.e., uniformly distributed) noise field. At frequencies where the spacing between adjacent elements is about half a wavelength, $\underline{M}_N \approx \underline{I}$ and the gain approximates K as discussed above. At lower frequencies, the array becomes superdirective and the improvement over conventional processing is quite striking. However, its performance is very sensitive to inaccuracies.

It is instructive to consider results when different optimisation criteria are used. For narrowband signals in gaussian noise, Edelblute *et al* [9] have shown that we have the same processor (i.e., the same set of weights $\underline{w}$) whether we maximise array gain, whether we minimise signal distortion in signal waveform estimation, or whether we derive the optimum detector (i.e., the likelihood ratio detector). The same processor also results from maximum likelihood estimation of certain signal parameters-for example, direction of arrival [10].

For broadband signals, however, there are differences using different criteria: the weights take the from

$$\underline{w}(f) = A(f)\underline{M}_N(f)^{-1}\underline{u}(f) \qquad (24)$$

where f = frequency, and A(f) defines a filter which depends on the spectra of signal and noise. The form of A(f) depends on the actual optimisation criterion used [9].

4.2 Constrained Optimal Beamforming

The unconstrained optimal beamformer uses prior knowledge of the noise cross-spectral matrix ($\underline{M}_N$) to suppress the noise power out of the array. Unfortunately, $\underline{M}_N$ is not usually known and needs to be estimated by measuring the array cross-spectral matrix when signal is absent. However, if the estimate of $\underline{M}_N$ inadvertently contains a contribution due to the wanted signal, then the signal itself will be treated as noise and will also be suppressed.

Capon [11] has devised a beamformer, akin in some respects to the unconstrained optimal beamformer, which overcomes this difficulty. It has proven very effective in practice and is now widely used. The technique in effect selects the set of weights $\underline{w}$ in such a manner that the power out of the beamformer is minimised, while the polar response in the direction of the desired signal is constrained to unity. In other words, $\underline{w}$ is selected so as to minimise $E(\underline{w}^H\underline{x}\underline{x}^H\underline{w}) = \underline{w}^H\underline{R}\underline{w}$, subject to $\underline{w}^H\underline{u} = 1$, where $\underline{R} = E(\underline{x}\underline{x}^H)$ is the cross-spectral matrix of the noise plus signal (not normalised).

An attack on this problem using a Laplace multiplier is straightforward and yields

$$\underline{w} = \underline{R}^{-1}\underline{u}/(\underline{u}^H\underline{R}^{-1}\underline{u}). \qquad (25)$$

The power out of the array is

$$\underline{w}^H\underline{R}\underline{w} = (\underline{u}^H\underline{R}^{-1}\underline{u})^{-1}.$$

Capon's technique is widely known as the Maximum Likelihood Method, a term that is arguably a misnomer because its derivation differs from that normally used for maximum likelihood estimation. As with the unconstrained optimal beamformer, any

signal received from a direction off the exact steered direction is regarded as interference and tends to be suppressed. In this way, this processor provides high resolution. Needless to say, one has therefore to be particularly careful that the beam-steered direction is in fact that of the desired signal, lest that signal be suppressed. To make the beamformer less sensitive to perturbations in signal direction, several authors ,e.g., [12,13], have suggested imposing equality constraints to broaden the main lobe of the polar diagram. A compromise is then struck between resolution and sensitivity.

Constraints can also be imposed on the magnitude of the weighting vector $\underline{w}$ in a number of ways, such as $\Sigma|w_i| < c$ or $|w_i| < c$ (where c is some constant). By doing this the sensitivity to internal noise and to errors (such as an error in the steered direction) can be reduced.

5 ESTIMATORS

The processors considered so far rely on reducing external noise and interference by controlling the polar diagram - i.e., by generation of nulls steered in appropriate directions, and low sidelobes. Implicit in this approach is the assumption that we know the signal arrival direction. This assumption can be valid in, for example, point-to-point radio communications, but in many other applications such as passive sonar we do not usually know the signal direction - perhaps, not even whether a signal is present at all. What is often done in cases where it is required to detect a signal whose direction is unknown is to estimate signal strength versus direction: an increase in estimated signal strength in a particular direction can indicate the presence of a signal source. Much theoretical effort in array processing is therefore directed towards the search for better estimators of signal angular distribution.

5.1 Linear Estimators

The beamforming techniques discussed in the preceding sections can all be used as estimators: the power output from a beam (obtained by squaring the modulus of the output of the beamformer) gives some estimate of the signal strength received from the beam direction. By generating many such overlapping beams, covering all directions of interest, we can estimate the distribution of signal strength versus direction.

These beamformers are all linear estimators of power, in the sense that the contributions from individual signals add linearly. In general, all the techniques discussed so far share one weakness: the effect of a signal source away from the beam direction is to cause an increase in the estimate (unless the direction of the former happens to coincide with a null in the polar diagram), so the estimate is biased positively. A similar bias is produced by receiver noise.

5.2 Non-Linear Estimators

Improvements in the performance of estimators are possible if we do not confine ourselves to linear estimators.

Here we address the case in which we know or can assume the directions of arrival of signals. We set up a model in which there are N random signal sources in known directions; the signal from any one source arrives at the receivers with known time differences, and the total signal received by one receiver is simply the sum of the individual contributions from all the sources. In practice, one rarely knows the directions of all the sources; what is done is to make N large, and to assume that the sources are distributed (either equally or unequally) over the arcs of interest. We confine ourselves to cases in which $N \geq K$. We outline briefly here the approach, which is covered in detail in [2].

At a given fequency, let y_n be the complex spectral component of the signal arriving from the n-th source at some reference point, and let $\underline{y} = [y_1 \ldots y_N]^T$.

We assume that the sources are random and independent of one another:

$$E(y_j y_m) = s_j \delta_{jm}. \tag{27}$$

and define ϕ_{mn} = the phase of the signal from the n-th source arriving at the m-th receiver

$v_{mn} = \exp(i\phi_{mn})$,

$\underline{V}$ = $[v_{mn}]$ (a KxN matrix) and , as before,

x_k = the complex spectral component of the signal received by the k-th receiver.

It is not difficult to see that

$$\underline{x} = \underline{V}\underline{y}. \tag{28}$$

The cross-spectral matrix of the receiver outputs is then

$$\begin{aligned} \underline{R} &= E(\underline{x}\underline{x}^H) \\ &= \underline{V}\, E(\underline{y}\underline{y}^H)\, \underline{V}^H \\ &= \underline{V}\underline{S}\underline{V}^H, \end{aligned} \tag{29}$$

where $\underline{S} = [s_j \delta_{jk}]$ is a diagonal matrix whose diagonal elements are the signal strengths which we wish to estimate.

Given the observed cross-spectral matrix $\hat{\underline{R}}$, what we do is to select the N diagonal elements of $\underline{S}$ so as to match $\underline{R} = \underline{V}\underline{S}\underline{V}^H$ to $\hat{\underline{R}}$ in some sense. Several new processors have been derived using this simple model; these are summarised below.

Least-Squares Fit Estimator:
Select $\underline{S}$ to minimise $\|\underline{R}-\hat{\underline{R}}\|^2$. [6]

Comprises the conventional beamformer followed by further processing.

Diagonal Fit Estimator:
Let Q be any hermitian symmetric matrix such that $\underline{V}\underline{Q}\underline{V}^H = \hat{\underline{R}}$. Select that $\underline{Q}$ for which the sum of the off-diagonal elements has the least-square value. Use the diagonal elements $\{q_{nn}\}$ as the estimates of the diagonal elements of $\underline{S}$.

Comprises the minimum bias beamformer (see section 3.1) followed by further processing.

---

6 $\|\ \|$ denotes the Euclidean norm of a matrix.

Inverse Fit Estimator:

Make $\hat{\underline{R}}^{-1}\underline{R} \simeq \underline{I}$, by minimising $|\hat{\underline{R}}^{-1}\underline{R}|$, subject to the constraint that $\mathrm{Tr}(\hat{\underline{R}}^{-1}\underline{R}) = K$.

Comprises the Capon 'maximum likelihood' beamformer (section 4.2) followed by further processing.

Maximum Likelihood Estimator:

For signals with a normal distribution, make $\underline{R}^{-1}\hat{\underline{R}} \simeq \underline{I}$, by minimising $|\underline{R}^{-1}\hat{\underline{R}}|$, subject to the constraint that $\mathrm{Tr}(\underline{R}^{-1}\hat{\underline{R}}) = K$.

Degenerates to a linear estimator when N=K (see section 3.2).

5.3 Deconvolution

Suppose we have a directional receiver which is physically rotated and pointed in different directions to estimate signal angular distribution. Because of the finite width of the beam and the presence of sidelobes, the estimate of signal strength in one direction will be biased by signals in other directions. The resultant estimate will in fact be the result of convolving the true signal angular distribution with the receiver polar diagram. If we know the latter, we can in principle use a deconvolution process to minimise the effect of the polar diagram. This technique was first applied decades ago by Bracewell and Roberts [14] to the case described above, in which the shape of the polar diagram is independent of steered direction. It can be extended to the general case of a beamsteered array of arbritrary geometry, the shape of whose polar diagram varies as the steered direction changes.

Another well-known deconvolution process is based on removal of the effects of strong interference falling in the sidelobes by estimating the interference strength (from a beam steered at it), computing the polar response in the direction of the main beam, and subtracting that computed response from the main beam output. This process is then repeated for any other strong source which may be present; it is useful when used to remove bias caused by a few large interferences.

We illustrate these deconvolution techniques by reporting work by Gray [15] who has studied a process using the complex outputs of an array steering N beams. Given these N conventional beamformer outputs as a first estimate of the signal distribution vector $\underline{y}$, the problem is to refine the estimate. The assumptions and terminology are as in section 5.2; from equation 28, $\underline{x} = \underline{V}\underline{y}$. The output of the conventional beamformer is

$$\underline{y}_o = \underline{V}^H\underline{x}/NK \quad ^7. \tag{30}$$

Then if $\underline{y}_j$ is the signal distribution vector after the j-th iteration, the array output would be $\underline{x}_j = \underline{V}\underline{y}_j$, and the corresponding output of the conventional beamformer would be $\underline{W}\underline{y}_j$, where $\underline{W} = \underline{V}^H\underline{V}/NK$. The error in the estimate is then

$$\underline{\varepsilon}_j = \underline{y}_o - \underline{W}\underline{y}_j.$$

The iterative algorithm is

$$\underline{y}_{j+1} = \underline{y}_j + \underline{\varepsilon}_j = \underline{y}_o + (\underline{I} - \underline{W})\underline{y}_j. \tag{31}$$

7 Gray used NK instead of the more usual K in the denominator to ensure convergence of his processing algorithm.

The iterations converge to the minimum bias estimator of section 3.1. Gray has also investigated a second iterative deconvolution process based on beam powers, which interestingly converges to the least-squares estimator of section 5.2.

5.4 Eigenvector Technique

Here we briefly describe a different approach to the problem of estimation of weak signal strength in the presence of strong interference. Consider an array whose receiver noises are independent of one another, in the presence of only a single signal. The cross-spectral matrix of the receiver outputs is then

$$\underline{R} = \mu\underline{u}\underline{u}^H + \nu\underline{I}. \tag{32}$$

It is easy to show that the largest eigenvalue of $\underline{R}$ is $(\mu K+\nu)$, with $\underline{u}/K$ as the corresponding eigenvector. A similar result obtains for an array with one interfering arrival much stronger than all the others.

Under these circumstances it is possible to estimate the dominant eigenvalue and its corresponding eigenvector and subtract their contributions from $\underline{R}$, effectively removing the effect of the interference. The weaker signals can then be estimated from the resultant matrix.

The technique involves computing the largest eigenvalue (λ_{max}) and its corresponding normalised eigenvector ($\underline{p}$), and then selecting that signal wavevector $\underline{k}_1$ such that the Euclidean distance between the corresponding signal vector $\underline{u}(\underline{k}_1)$ and $K^{\frac{1}{2}}\underline{p}$ is minimised. $\underline{u}(\underline{k}_1)$ then provides an estimate of signal direction, and λ_{max} an estimate of signal strength.

This strong signal can be excised by subtracting $\lambda_{max}\underline{u}(\underline{k}_1)\underline{u}(\underline{k}_1)^H$ from $\underline{R}$. If another predominantly strong signal now appears in the difference

$$\underline{R}_1 = \underline{R} - \lambda_{max}\underline{u}(\underline{k}_1)\underline{u}(\underline{k}_1)^H,$$

one can repeat the process on $\underline{R}_1$. In this way, the influence of strong signals on the cross-spectral matrix can be removed (or at least substantially reduced) iteratively. The resulting matrix can then be used in any of the processors (e.g., optimal, maximum likelihood method) to detect weaker signals.

6 APPLICATION OF MODERN SPECTRAL ESTIMATION TECHNIQUES

Just as signals from unknown directions can be detected by first estimating signal strength as a function of angle (see section 5), so signals of unknown frequency can be detected by estimating the power spectrum and then searching for lines. Spectral estimation techniques have in recent years evolved rapidly, for this and other applications. As mentioned earlier, by using the analogy between frequency and wavenumber such techniques can be applied to array processing. Some of the more important of these techniques are discussed in this section, with reference to their application to array processing where relevant.

6.1 Pole-Zero Modelling

A major thrust in spectral analysis research has been in the use of models which assume that the waveform whose spectrum is to be estimated has been

obtained by passing zero-mean white noise through a time-invariant linear filter, as shown in fig. 13. The transfer function of the filter is estimated, and hence the power spectrum of the waveform. Such models can be applied directly to the processing of signals from equispaced linear arrays, but in most cases much work remains to be done to extend them to arbitrary array geometries. In what follows, we outline the techniques as they are used for spectral analysis of a time series; the usual analogy can be drawn for estimating signal angular distribution using an array.

Figure 13 Pole-zero model.

If the input to the filter consists of the noise sequence $\{y_n\}$, then the output sequence $\{x_n\}$ is given by

$$x_n = \sum_{j=0}^{q} b_j\, y_{n-j} - \sum_{i=1}^{p} a_i\, x_{n-i}. \qquad (33)$$

Taking the z-transform of equation 33, we find that the transfer function H(z) of the filter is rational

$$H(z) = B(z)/A(z), \qquad (34)$$

where A(z) and B(z) are polynomials of order p and q respectively,
z^{-1} = $\exp(-i2\pi f\Delta t)$ is the unit delay operator
and Δt = step delay.

The estimated power spectrum is given simply by

$$P(x) = H(z)\, H(1/z). \qquad (35)$$

The estimation techniques can be divided broadly into three classes, depending on the type of filter used:

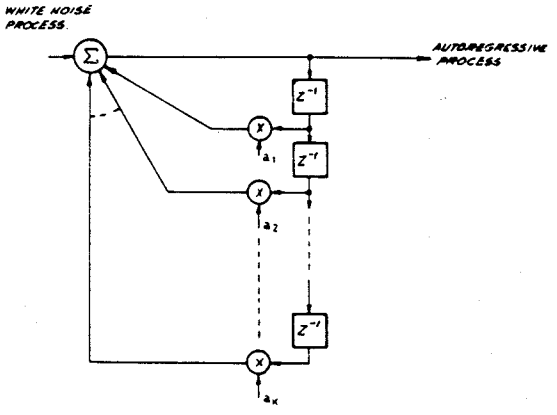


Figure 14 Autoregressive (AR) process.

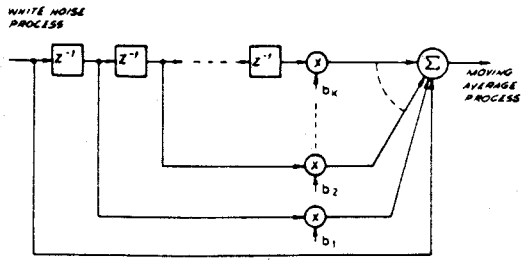


Figure 15 Moving average (MA) process.

i) All-pole or autoregressive (AR) model, in which $H(z) = 1/A(z)$, and $x_n = y_n - \sum_1 a_i\, x_{n-i}$; uses recursive filter with feedback paths as shown in fig. 14.

ii) All-zero or moving average (MA) model, in which $H(z) = B(z)$, and $x_n = \sum_j b_j\, y_{n-j}$; uses feedforward paths as shown in fig. 15.

iii) General pole-zero or autoregressive-moving-average (ARMA) model, in which $H(z) = B(z)/A(z)$ and equation 33 applies.

Because the AR model comprises a set of resonant filters, it is capable of high resolution and has therefore received much attention.

6.2 Maximum Entropy Spectral Estimation

If we are given the complete autocovariance function $\{r(n)\}$ of a stationary time series, where $r_n = E\{x_m^* x_{m+n}\}$, then its power spectrum is given by

$$\Phi(f) = \sum_{-\infty}^{\infty} r(n) \exp(-i2\pi fn\Delta t), \qquad (36)$$

where Δt is the interval between samples.

In practice, we have only a finite observation period, so in conventional spectral estimation we use some estimator such as

$$\Phi(f) = \sum_{-N}^{N} \alpha(n)\, r(n) \exp(-i2\pi fn\Delta t), \qquad (37)$$

where $\alpha(n)$ is some suitable weighting function.

In equation 37, we make the implicit assumption that the autocovariance function is identically zero outside the observation period. (By analogy, for a linear array of equispaced receivers, we make the assumption, for the narrowband case, that the spatial autocovariance function is zero for distances greater than the length of the array.)

In a classic paper, Burg [16] pointed out that such an assumption is unreasonable. There are infinitely many band-limited spectra $\Phi(f)$ whose autocovariance is identically equal to the finite set of observed $\{r(n)\}$:

$$r(n) = \int_{-B/2}^{B/2} \Phi(f) \exp(i2\pi fn\Delta t)\, df, \quad n = -N, \ldots 0, \ldots N. \qquad (38)$$

Burg argued that there was a rationale in selecting that spectral estimate which made the least assumptions regarding the behaviour of the time series outside the observation period, while remaining consistent with the observations. Such a spectrum is the most random, or has maximum entropy.

Specifically, he maximised the entropy

$$H = \int \ln \Phi(f)\, df \qquad (39)$$

subject to equation 38.

The resulting maximum entropy method (MEM) has received much attention in the literature. It has been shown [17] that the MEM fits, in a least-squares sense, an AR process of order K to the observed data. (For array processing, K would be the number of receivers.) It is capable of high resolution and has been found to be particularly useful for estimating spectra from short samples of data. Practical considerations often constrain the

number of receivers which can be put into an array so the MEM is valuable because of the high resolution it provides.

Let us denote the power spectrum estimated by an m^{th} order AR process by $P_{AR}^{(m)}(f)$. It has been shown [18] that the power spectrum estimated using Capon's maximum likelihood method (see section 4.2), $P_{ML}(f)$, is in fact related to the AR estimate as follows:

$$\frac{1}{P_{ML}(f)} = \frac{1}{K} \sum_{m=1}^{K} \frac{1}{P_{AR}^{(m)}(f)} \qquad (40)$$

where both $P_{ML}(f)$ and $P_{AR}^{(m)}(f)$ are calculated using the same autocovariance matrix of order K.

6.3 Linear Predictive Spectral Estimation

Wiener linear prediction techniques can also be used to estimate spectra. In this case, the applicability of the technique does not rely on the frequency-time/wavenumber-space analogy, and so the technique is not confined to equispaced linear arrays; it can readily be applied to arrays of arbitrary geometry.

Suppose we wish to estimate the spectral output of the L-th receiver of an arbitrary array, from a linear combination of the spectral outputs of the remaining K-1 receivers. In other words, we have

$$\hat{x}_L = - \sum_{m \neq L} w_m x_m \qquad (41)$$

One criterion for selecting the set of weights $\{w_m\}$ is the minimise the mean-square error between our estimate $\hat{x}_L$ and our observation x_L; i.e., we minimise

$$\varepsilon = E(|\hat{x}_L - x_L|^2). \qquad (42)$$

It is convenient to define a vector $\underline{w}$ whose entries areall arbitrary except $\underline{w}_L = 1$. Then we have

$$\varepsilon = E(|\underline{w}^H \underline{x}|^2)$$

$$= \underline{w}^H \underline{R}\, \underline{w}, \qquad (43)$$

where $\underline{R} = E(\underline{x}\underline{x}^H)$ as before.

The problem then is to minimise ε subject to the constraint that $w_L = 1$, or equivalently, that

$$\underline{e}_L^T \underline{w} = 1, \qquad (44)$$

where $\underline{e}_L$ is defined to be a vector whose L-th element is unity and with all other elements zero. The solution is readily obtained using a Laplace multiplier:

$$\underline{w} = \frac{\underline{R}^{-1}\underline{e}_L}{\underline{e}_L^T \underline{R}^{-1} \underline{e}_L} .$$

When this technique is applied to spectrum estimation, the estimated power spectrum is given by the mean-square error (ε) divided by the power spectrum of the predictor coefficients. Johnson [19] argues that by analogy the estimate of signal distribution is

$$P_{LP}(\underline{k}) = \frac{\underline{w}^H \underline{R} \underline{w}}{|\underline{w}^H \underline{v}(\underline{k})|^2}$$

$$= \frac{\underline{w}^H \underline{R}^{-1} \underline{w}}{|\underline{e}_L^T \underline{R}^{-1} \underline{v}(\underline{k})|^2} \qquad (45)$$

Note that L is arbitrary - i.e., we may select any one of the receivers. It can be argued that the 'best' choice of L is that which minimises the mean-square prediction error, which is given by the reciprocal of the smallest diagonal element of R^{-1}. However, it appears that the choice is by no means clear in practice [19].

7 ADAPTIVE PROCESSING

The processing techniques described above require the computation of a weight vector $\underline{w}$ on the basis of *a priori* knowledge of the ambient noise environment of the array. The assumption is also implicitly made that the noise field properties do not change with time, so that the weight vector is also time-invariant.

Although not unknown, the assumption of complete *a priori* knowledge of the noise environment of a receiving array is rarely valid in practice, so it is usually necessary to adapt the weights in some way to the environment. Some of the most significant advances in array processing in recent years have been in the field of adaptive processing. We outline here two of the more important techniques.

7.1 Adaptive Weight Vector Update

Widrow *et al* [20,21] pioneered an approach in which the weights are iteratively adapted to minimise the least-mean-square error between the actual array output signal and some desired signal. They operated in the time domain, but the technique can equally be applied in the frequency domain.

Exact minimisation of the least-mean-square error involves solution of the Wiener-Hopf equation, requiring inversion of the KxK covariance matrix, which is computationally demanding for an adaptive processor. Instead, Widrow used a steepest decent algorithm to minimise the instantaneous error, thus neatly avoiding the need to store and invert the covariance matrix. However, the technique involves the injection of a pilot signal to simulate the signal to be detected, when it is known that signal is absent.

Subsequently, Frost [22] derived an adaptive processor which overcomes the need for a pilot signal. While Widrow's algorithm is an adaptive implementation of the optimal beamformer (section 4.1), Frost's derives from Capon's MLM Beamformer (section 4.2). The problem is posed as adjustment of the weight vector $\underline{w}$ to minimise the array output power $E\{|Y(\underline{k})|^2\} = \underline{w}^H \underline{R} \underline{w}$, using a steepest descent technique, subject to the constraint that

$$\underline{u}^H \underline{w} = 1. \qquad (46)$$

The exact solution, given earlier in equation 25 involves $\hat{R}^{-1}$. However Frost (like Widrow), uses $\underline{x}\underline{x}^H$ as an approximation to $\underline{R}$, thereby greatly simplifying the computations.

In what follows, we use subscripts j to denote values at the j^{th} interation. Using some appropriate initial weight vector such as $\underline{w}_1 = \underline{u}$,

8 Actually, Frost addressed the more general case in which a number of constraints are applied simultaneously, but for simplicity we impose only a single constraint. Also, whereas he operated in the time domain, we work here in the frequency domain, using notation consistent with that of the rest of this paper.

the resulting algorithm at the j^{th} iteration is

$$\underline{w}_{j+1} = \underline{P}\{\underline{w}_j - \alpha\gamma^*(\underline{k})\underline{x}_j\} + \underline{u}/K, \qquad (47)$$

where α is a scalar determining the rate of convergence of the weights, and $\underline{P} = \underline{I} - \underline{u}\underline{u}^H/K$ is a projection operator.

It can be shown that the process converges if $0 < \alpha < 2/\lambda_{max}$, where λ_{max} is the largest eigenvalue of $\underline{PRP}$.

The power out of the beamformer can be represented in terms of components due to individual eigenvalues of $\underline{PRP}$. In the case where a single strong interfering source dominates the ambient noise field, output power and convergence rates can be relatively easily predicted. Some results for experimental sonar data are presented in reference 23, where good agreement is shown between predicted and actual convergence rates.

7.2 Matrix Inverse Update

As mentioned earlier, the exact solution for the weight vector $\underline{w}$ - whether for the unconstrained or constrained optimal beamformer - involves the inverse of the array cross-spectral matrix $\underline{R}$. Widrow and Frost avoid inverting the matrix by using $\underline{x}\underline{x}^H$ as an approximation to $\underline{R}$.

However, it is possible to adapt $\underline{R}_j^{-1}$ as each new observation $\underline{x}_j$ is made, using Woodbury's identity [24]:

$$\text{if} \quad \underline{R}_{j+1} = (1-\alpha)\underline{R}_j + \alpha\underline{x}_j\underline{x}_j^H, \qquad (48)$$

$$\text{then} \quad \underline{R}_{j+1}^{-1} = \frac{1}{(1-\alpha)}\left\{\underline{R}_j^{-1} - \beta\,\underline{q}\underline{q}^H\right\}, \qquad (49)$$

$$\text{where} \quad \underline{q} = \underline{R}_j^{-1}\underline{x}_j \ ,$$

$$\text{and} \quad \beta = \{\underline{x}_j^H\underline{R}_j^{-1}\underline{x} + (1-\alpha)/\alpha\}^{-1}.$$

The beamformer weights at each iteration can readily be calculated for a particular steered direction $\underline{u}$, using equation 25 . Note that (a) it is not necessary to compute and store $\underline{R}_{j+1}$, and (b) the same $\underline{R}_{j+1}^{-1}$ is used for all the beams to be generated.

Lunde [25] discusses both algorithms and compares their performance in a particular environment and concludes that in some cases matrix update outperforms gradient descent quite convincingly.

8 DISCUSSION

8.1 Choice of Processing Technique.

With the large choice of techniques, the reader may well be left bemused: which of the many available options is best? Which should one use?

Regrettably, there are no simple answers. In computer studies, using artificially-generated data, it is found (to no one's surprise) that best results are obtained when the processor model is the same as the model used to generate the data. For example, an AR technique gives good results when processing data using the same AR model. However, if the data are generated using an AR model of order p, spurious peaks can result from a processor using an AR model of order greater then p. Thus even if it is known (or can be assumed) that the observed data fit an AR model (say), it may be necessary to estimate its order. Considerable effort has been devoted to studies in this area [26].

Another major factor in selecting a processor is the extent of - and confidence in - one's knowledge of the physical world. We have given examples in which small differences between the model and the real world can result in serious degradation in performance: if a beam is steered in the wrong direction using the maximum likelihood method, it will tend to suppress the wanted signal; similarly, if the wavefront is assumed to be plane but happens instead to be slightly curved, it will be discriminated against. The more advanced processors, which seek to increase performance, seem invariably to do so at the sacrifice of robustness. In this context, adaptive beamformers are attractive, as they can be designed to adjust their parameters to prevailing conditions. Matrix inverse update processors are of particular interest to this problem area, especially when used to implement constrained optimisation methods. These are likely to be the subject of continuing research.

In recent years, there have been significant developments in array processing using modified spectral analysis techniques. However, it can be questioned whether a model which is successful in representing the spectrum of a time series will be equally relevant in representing the angular distribution of signal sources. For example, there are situations in which the one signal can travel to the array via two or more paths, with some of the received multipath signals having some degree of coherence with one another. Such coherent arrivals have been given scant attention, and there appears to be scope here for further study.

In our experience, it is essential to test proposed signal processing techniques using experimental data. It is only then that some understanding can be gained of appropriate models.

Of course, in the final choice of technique there are many other practical considerations, such as cost, complexity and reliability, and whether one has the luxury of off-line processing or is constrained to producing results in real-time; but these considerations fall outside the purview of this paper.

8.2 Array Geometry

Another main area of difference between spectral analysis and array processing is in the effect of array geometry. It has long been known that, from the point of view of performance, symmetry in array geometry is in general undesirable[9]. (An extreme example of a symmetric array is the linear array, which is incapable of distinguishing between signals arriving at the same angle to the axis of the array.) However, we know of no systematic technique to design an array so as to minimise this effect. The type of problem that one encounters in practice is to design an array with constraints on dimensions and/or on the number of elements. Here, also, there appears to be scope for further research.

One occasionally encounters the situation in which the geometry of the array is not precisely known. The challenge then is to process the array signals

9 On the other hand, a symmetrically disposed array often makes signal processing easier to implement in practice, because some functions are rendered repetitive.

in such a manner as to infer both the angular distribution of signal sources and the array geometry. Although work is continuing in this field, it, too falls outside the scope of this paper.

8.3 The Future

It is likely that, in the immediate future, the present trend of adapting developments from spectral analysis to array processing will continue. As indicated earlier, much work is still needed to adapt some of the spectral analysis techniques to arrays of arbitrary geometry. The fruit of some explorations into this area can be expected shortly.

We have not mentioned many of the new spectral analysis techniques, which use other models - for example, a number of spectral lines of unknown amplitude and frequency in a continuous noise background. When adapted for array processing, each of these techniques will need to be tested using actual experimental data.

In the longer term, it is possible that worthwhile achievements may result from the collaboration of researchers into the environment (both signal propagation and noise) with researchers into array processing. The models generated should then be more closely attuned to the physical environment, taking into account such factors as coherence between multipath arrivals.

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